

## Formula Sheet: Math 2110

- Projections of  $\mathbf{b}$  onto  $\mathbf{a}$

$$\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$$
$$\text{proj}_{\mathbf{a}}\mathbf{b} = \left( \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|} \right) \frac{\mathbf{a}}{|\mathbf{a}|}$$

- Cross product of  $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$  and  $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

- Theorem on cross and dot products of vectors  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$ .

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$$
$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

- Volume of parallelepiped determined by  $\mathbf{a}$ ,  $\mathbf{b}$ , and  $\mathbf{c}$  is  $|\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}|$ .
- Derivatives of dot and cross products

$$\frac{d}{dt}(\mathbf{u}(t) \cdot \mathbf{v}(t)) = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t)$$
$$\frac{d}{dt}(\mathbf{u}(t) \times \mathbf{v}(t)) = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)$$

- Curvature is given by

$$\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

- The binormal is  $\mathbf{B} = \mathbf{T} \times \mathbf{N}$ .
- Acceleration is  $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$  where

$$a_T = \frac{dv}{dt} = \frac{\mathbf{r}'(t) \cdot \mathbf{r}''(t)}{|\mathbf{r}'(t)|}$$
$$a_N = \kappa v^2 = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}$$

- The discriminant in the second derivative test for two-variable functions is given by

$$D = f_{xx}(a, b)f_{yy}(a, b) - \left(f_{xy}(a, b)\right)^2$$

- Green's theorem: For a smooth positively oriented simple closed curve  $C$  enclosing a region  $D$

$$\int_C P dx + Q dy = \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$